

DUBLIN INSTITUTE OF TECHNOLOGY

First Year Engineering Entrance Examination 2015

In

MATHEMATICS

April 2015

Attempt any 6 of the following 8 QUESTIONS

Time Allowed: 3 hours

Each question has 100 marks

All question carry equal marks

Maths Tables and graph paper are available for use

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1. (a) Make x the subject of the formula: (30)

$$y = \frac{ax^2}{4b} + 5b$$

- (b) Factorize $y = x^2 + 4x + 4$ and $z = x^2 - 5x + 6$. Show the roots on a graph. (20)

- (c) Simplify the following expression involving indices: (20)

$$\frac{x^2yx + x^3y^3 - y^2x^5y^4}{x^2y^2}$$

- (d) Solve the simultaneous equations: (30)

$$\begin{aligned}x + 2y + 2z &= 1 \\2x + z &= 2 \\x + y + z &= 3\end{aligned}$$

2. (a) Find values of the first derivatives of the following at the given points:

(i) $f(x) = \sin(3x) \cos(2x)$ at $x = \pi/3$ (25)

(ii) $g(x) = \sqrt{5x^3 + 2x^2 + 2x + 25}$ at $x = 0$ (25)

- (b) Given the function $y = 2x^3 - 6x - 1$. Find the two turning points and specify if they are maximum or minimum points. (25)

- (c) Given the function $y = ke^x \sin(x)$ and that $\frac{dy}{dx} = 2$ for $x = 0$, find the value of k . (25)

3. (a) Solve for x :

(i) $\log_{10}(x + 1) - 2 \log_{10}(x) = 1$ (10)

(ii) $\ln\left(\frac{x+1}{2x}\right) = 3.4$ (15)

(b) In a chemical reaction, the amount of starting material in cm^3 left after t hours is given by: (25)

$$M = 15e^{-0.3t}$$

(i) What is the initial amount of M ?

(ii) How much material is left after 3 hours and estimate how long it will take for M to fall to 5 cm^3 .

(c) The following table gives measurements of temperature T , in $^{\circ}\text{C}$, at various times t minutes. The values of T are believed to be related to time t by the law $T = Ae^{kt}$.

t in minutes	5	10	15	20	25
T $^{\circ}\text{C}$	121.3	73.6	44.6	27.1	16.4

(i) Use the log-linear graph paper supplied to confirm this for the given table of values. (20)

(ii) Find the values of A and k . (30)

4. (a) Given $z = 2 - 4i$ and $h = -2 - 2i$. Calculate $z + h$, $z - h$, $z \times h$ and $\frac{z}{h}$. (25)

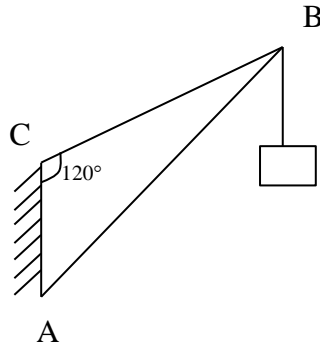
(b) Show that $z = 2 + i$ is a root of $z^3 + z^2 + 2z - 9 - 17i = 0$. (25)

(c) Express $(2 - 4i)$ in polar form and calculate $(2 - 4i)^4$. (25)

(d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form: $5i$, $-2 - 2i$, $3 - 5i$, $-1 + 7i$, 2 and $1 + i$. (25)

5. (a) The figure below represents a jib crane. AB is the inclined jib of the crane and is 10m long. AC is 4m long. Determine: (35)

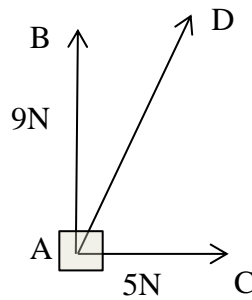
- (i) The inclination of the jib to the vertical.
(ii) The length of CB .



- (b) Consider an object at A acted upon by forces AB and AC as shown below. Force AB is 9 N, force AC is 5N and they act at right-angles to one another. (30)

Calculate:

- (i) The magnitude (length $|AD|$) of the resultant force AD .
(ii) The direction (angle $\angle DAC$) of the resultant force AD .



- (c) Sketch the graphs of $\cos(x)$ and $2\cos(x)$ between 0 and 2π . Hence solve the following equation $\cos(2x) = 0.866025$ for $0 \leq x \leq 360^\circ$. (35)

6. (a) The points $A(1,3)$, $B(1,7)$ and $C(7,7)$ lie on the circle $(x - 4)^2 + (y - 5)^2 = 13$.
- (i) Determine the radius and centre of the circle. (10)
- (ii) Draw a sketch of the circle and points and show that AC is a diameter of the circle. (20)

- (b) The following values of resistance R in ohms and corresponding voltage V volts are obtained from a test on a filament lamp:

R ohms	30	46.25	75	105	127.5
V volts	16	29	52	76	94

- (i) Plot a graph with R in the vertical axes and V in the horizontal axes. (10)
- (ii) Determine the gradient of the graph, the R axis intercept and the equation of the graph. (15)
- (iii) What is the value of R when $V = 110$ volts? (5)
- (c) Find the equation of the line that passes through the point of intersection of the lines $2x - 4y - 6 = 0$ and $x + 3y - 8 = 0$ and is parallel to the line $3y + 3x = 4$. (40)

7. (a) Determine the following integrals:

(i) $\int \left(3x^7 + 3x^{\frac{2}{3}} - \frac{1}{x^3} \right) dx$ (25)

(ii) $\int (2x + 3) \cos(x^2 + 3x + 8) dx$ (25)

(iii) $\int \frac{x^4 + 4x^3 - 2x^2 + x - 4}{x^2} dx$ (25)

- (b) Find the area under the curve $y = x^3 + 3x + 3$ between the values $x = 0$ and $x = 3$. (25)

8. Given:

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 1 \\ 5 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 5 \\ 7 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 4 & 3 \\ 1 & 1 \\ 7 & 6 \end{pmatrix}$$

(a) Calculate the following: $A + D$, $A - D$ and $A \times B$. (25)

(b) Can you perform the following calculations: $\times A$, $C \times A$, $A \times C$ and $A \times D$? Explain your reasoning for each. (25)

(c) Find the inverse of B . (20)

(d) Given $E = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find a constant k that fulfils the following equation: (30)

$$E \cdot E^{-1} + \begin{pmatrix} k+1 & 2k \\ k & k \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix}$$