

DUBLIN INSTITUTE OF TECHNOLOGY

First Year Engineering Entrance Examination 2016

In

MATHEMATICS

April 2016

Attempt any 6 of the following 8 QUESTIONS

Time Allowed: 3 hours

Each question has 100 marks

All question carry equal marks

Maths Tables and graph paper are available for use

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1. (a) Make x the subject of the formula: (25)

$$y = \frac{3x}{4} - b + 2c$$

- (b) Factorize $y = x^2 - 4x$ and $z = x^2 - 6x + 9$. (25)

- (c) Simplify the following expression involving indices: (25)

$$\frac{x^3y^2x^2 + x^2y^2 - y^3x^6y^2}{x^5y^5}$$

- (d) Expand the following binomial: $(x + 2)^5$. (25)

2. (a) Find values of the first derivatives of the following:

(i) $f(x) = \sin(x)(3x^2 + 2x - 5)$ (25)

(ii) $g(x) = (3x^3 + 3x^2 - 2x + 3)^4$ (25)

- (b) Given the function $y = x^3 + 2x^2 - 6x + 10$. Find the two turning points and specify if they are maximum or minimum points. (25)

- (c) Given the function $y = k \sin(2x)$ and that $\frac{dy}{dx} = 4$ for $x = 0$, find the value of k . (25)

3. (a) Solve for x :

(i) $\log_{10}(x+2) - \log_{10}(x) = 1$ (20)

(ii) $\ln\left(\frac{x-1}{x+2}\right) = 2$ (20)

(b) In a chemical reaction, the amount of starting material in cm^3 left after t hours is given by $M = 35e^{-0.2t}$.

What is the initial amount of M ? How much material is left after 3 hours and estimate how long it will take for M to fall to 1 cm^3 . (20)

(c) The following table gives measurements of temperature T , in $^{\circ}\text{C}$, at various times t in minutes. The values of T are believed to be related to time t by the law $T = A e^{kt}$.

t in minutes	5	10	15	20	25
T $^{\circ}\text{C}$	96.301	123.654	158.775	203.871	261.776

(i) Use the log-linear graph paper supplied to confirm this for the given table of values. (20)

(ii) Find the values of A and k . (20)

4. (a) Given $z = 1 - i$ and $h = -1 - 3i$. Calculate $z + h$, $z - h$, $z \cdot h$ and $\frac{z}{h}$. (25)

(b) Given $z = 3 + i$, calculate z^4 , z^3 , z^2 and express the results in rectangular and polar forms. (25)

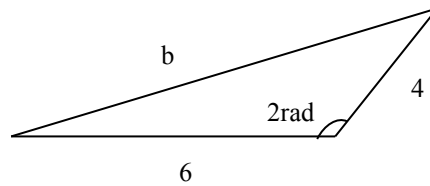
(c) Find a if: $(3 + i)^2 + 2 + 2ai = -5a + 2i$. (25)

(d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form: $5i + 2$, $1 - 2i$, $-5 - 5i$, $-1 + i$, $2i$ and $7 + 0i$. (25)

5. (a) Solve $\cos(2x) = -0.5$ for values of x between 0° and 360° . (25)

(b) The angle of elevation of a wind turbine from a point on the ground is 35° . The angle of elevation increases by 7° when measured from a second point also on the ground, but 25 m nearer the tower than the first point. Find the height of the wind turbine and the distance from the base of the turbine to the first and second points. (25)

(c) Find the value of b and the missing angles in the following triangle: (25)



(d) Sketch the graphs for $y = 2\sin(x)$ and $h = \sin(x)$. (25)

6. (a) Given the points $A(2,3)$, $B(10,7)$ and $C(7,0)$ and the circle $(x - 4)^2 + (y - 4)^2 = 25$.

(i) Determine the radius and centre of the circle. (10)

(ii) Calculate if the given points A , B and C lie inside, outside or on the circle. (20)

(b) Given the following values:

x	0	10	20	30	40
y	5	25	45	65	85

(i) Plot the graph of the function. What type of function is it? (15)

(ii) Determine the slope, the x and y intercepts and the equation of the function. (15)

(iii) What is the value of y when $x = 250$? (5)

(c) Find the equation of the line that passes through the point of intersection of the lines $x + y + 6 = 0$ and $x - 8 = 0$ and is perpendicular to the line $y - 2x = 2$. (35)

7. (a) Determine the following integrals:

$$(i) \int (6x + 3)(3x^2 + 3x)^3 dx \quad (25)$$

$$(ii) \int \left(\frac{2x^5 - x^{5/2} - x^{-2}}{x^2} \right) dx \quad (25)$$

$$(iii) \int (5x^3 - 4x^{-3} + \frac{12}{x} - 4e^{-2x}) dx \quad (25)$$

(b) Find the area under the curve $y = 2x^3 + 3x^2 + 2x + 20$ between the values $x = 0$ and $x = 3$. (25)

8. Given:

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 4 \\ 2 & 7 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 2 & 1 \\ 5 & 3 & 4 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 5 & 3 \\ 6 & 6 \\ 7 & 2 \end{pmatrix}$$

(a) Calculate the following: $A + B$, $A - B$ and $A \times B$. (30)

(b) Can the following calculations be performed? (30)

$$D \times A, \quad C \times A, \quad A \times C \quad \text{and} \quad A \times D$$

Explain your reasoning for each and show the size of the result matrix.

(c) Solve the following system of simultaneous equations: (40)

$$\begin{aligned} x + 2y + 2z &= 1 \\ 2x + y + 2z &= 2 \\ 3x + y + z &= 1 \end{aligned}$$