

DUBLIN INSTITUTE OF TECHNOLOGY

First Year Engineering Entrance Examination 2017

In

MATHEMATICS

April 2017

Attempt ALL 5 QUESTIONS

Time Allowed: 3 hours

All questions carry equal marks

Maths Tables and graph paper are available for use

Dr. Kevin Kelly

Mr. Kevin Gaughan

Ms. Marisa Llorens-Salvador

1. (a) Express $z = (2 - 4i)(3 - 3i)$ in polar form and calculate z^4 . Express the results both in polar and rectangular forms.
- (b) Find a if $z=2+i$ is a root of $z^2 + 3z + a - 14 - 7i = 0$.
- (c) Simplify the following expression involving indices:

$$\frac{x^4y^2x^3 + x^2y^3 - y^2x^5y^4}{x^2y^3}$$

- (d) Expand the following binomial: $(x - y)^5$
2. (a) A lot contains 12 items of which 5 are defective. Three items are drawn at random from the lot, one after another. Find the probability p that all three are non defective.
- (b) In a survey 100 students were asked about their preference for foreign language subjects. It was found that 44 chose German, 38 chose French and 40 chose Spanish. It was also found that 17 chose German and French, 9 chose French and Spanish, 12 chose German and Spanish and 10 didn't choose any foreign language.
- (i) Use a Venn diagram to show the number of students, their options and fill in the correct number of students in each of the regions of the diagram.
- (ii) Find the number of students that chose only one foreign language.
- (iii) Determine the number of students that chose 2 foreign languages.
- (c) The mean height of a group of 200 people is 165cm with a standard deviation of 10cm. Assuming the heights are *normally* distributed find the probability of a person's height being:
- (i) less than 168cm
- (ii) more than 165cm
- (iii) greater than 185cm

3. (a) Make x the subject of the formula: $y + 2xb = \frac{x}{4b} + x5b$

(b) Solve the simultaneous equations:

$$\begin{aligned} 3x + 2y + z &= 5 \\ x + 4y + 4z &= 4 \\ 2x + 3y + 2z &= 2 \end{aligned}$$

(c) Solve for x :

(i) $\log_{10}(x + 1) + \log_{10}(x - 1) = 2$

(ii) $\ln\left(\frac{x-1}{x-3}\right) = 2$

(d) In a chemical reaction, the amount of starting material in grams left after t hours is given by $M = 50e^{0.1t}$.

(i) What is the initial amount of M ?

(ii) How much material is present after 20 hours and estimate how long it will take for M to reach 500 grams.

4. (a) Given the following :

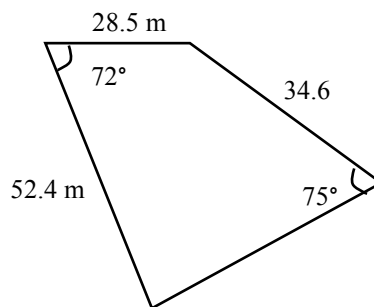
C_1 is the circle $x^2 + y^2 + 2x - 2y - 23 = 0$

C_2 is the circle $x^2 + y^2 - 14x - 2y + 41 = 0$

Prove that both circles touch externally and find the point of contact.

(b) Find the equation of the line that passes through the point of intersection of the lines $3x + 2y - 1 = 0$ and $2x - y + 7 = 0$ and is perpendicular to the line $3y + 3x = 4$.

(c) A building site is in the form of a quadrilateral as shown below. Determine the length of the perimeter of the site.



(d) Sketch the graphs of $\sin(x)$ and $\sin(2x)$ between 0 and 2π . Hence solve the following equation $\sin(2x) = 0.866025$ for $0 \leq x \leq 360^\circ$.

5. (a) Find values of the first derivatives of the following at the given points:

(i) $f(x) = (x^2 + 4x - 4)^3$ at $x = 1$

(ii) $g(x) = (5x^2 - 3x)(3e^{3x})$ at $x = 0$

(b) Given the function $y = x^3 - 5x^2 - x + 2$. Find the two turning points and specify if they are maximum or minimum points.

(c) Determine the following integrals:

(i) $\int \left(x^3 - 2x^2 + 3x^{-1} - \frac{2}{x^3} \right) dx$

(ii) $\int 2e^{2x} + \frac{2x^2 + 3x^4 + 4x + 5}{x^2} dx$

(d) Find the area under the curve $y = 2x^3 + 2x - 3$ between the values $x = 2$ and $x = 4$.