

DUBLIN INSTITUTE OF TECHNOLOGY

First Year Engineering Entrance Examination 2017

In

MATHEMATICS

August 2017

Attempt ALL 5 QUESTIONS

Time Allowed: 3 hours

All questions carry equal marks

Maths Tables and graph paper are available for use

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1. (a) Express $z = (1 - 2i)(3 - 3i)$ in polar form and calculate z^3 . Express the results both in polar and rectangular forms.
- (b) Find a if $z=1+i$ is a root of $z^2 + 2z + 2a - 12 + 4i = 0$.
- (c) Simplify the following expression involving indices:

$$\frac{x^6y^2x^3 + x^4y^3 - y^2x^7y^4}{x^2y^2}$$

- (d) Expand the following binomial: $(x - y)^5$
2. (a) A lot contains 14 items of which 4 are defective. Three items are drawn at random from the lot, without being replaced. Find the probability p that all three are non defective.
- (b) In a survey 100 students were asked about their preference for foreign language subjects. It was found that 42 chose German, 39 chose French and 44 chose Spanish. It was also found that 17 chose German and French, 9 chose French and Spanish, 12 chose German and Spanish and 10 didn't choose any foreign language.
- (i) Use a Venn diagram to show the number of students, their options and fill in the correct number of students in each of the regions of the diagram.
- (ii) Find the number of students that chose only one foreign language.
- (iii) Determine the number of students that chose 2 foreign languages.
- (c) The mean height of a group of 300 people is 167cm with a standard deviation of 8cm. Assuming the heights are *normally* distributed find the probability of a person's height being:
- (i) less than 169cm
(ii) more than 167cm
(iii) greater than 186cm

3. (a) Make x the subject of the formula: $y + 3xb = \frac{x}{4b} + x7b$

(b) Solve the simultaneous equations:

$$\begin{aligned} 3x + 2y + z &= 5 \\ x + 4y + 4z &= 4 \\ 2x + 3y + 2z &= 2 \end{aligned}$$

(c) Solve for x :

(i) $\log_{10}(x + 1) + \log_{10}(x - 1) = 2$

(ii) $\ln\left(\frac{x-1}{x-3}\right) = 2$

(d) In a chemical reaction, the amount of material in grams after t hours is given by

$$M = 30e^{0.1t}$$

(i) What is the initial amount of M ?

(ii) How much material is present after 30 hours and estimate how long it will take for M to reach 700 grams.

4. (a) Given the following :

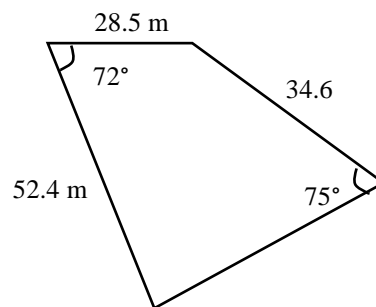
$$C_1 \text{ is the circle } x^2 + y^2 + 2x - 2y - 23 = 0$$

$$C_2 \text{ is the circle } x^2 + y^2 - 14x - 2y + 41 = 0$$

Prove that both circles touch externally and find the point of contact.

(b) Find the equation of the line that passes through the point of intersection of the lines $3x + 2y - 1 = 0$ and $2x - y + 7 = 0$ and is perpendicular to the line $3y + 3x = 4$.

(c) A building site is in the form of a quadrilateral as shown below. Determine the length of the perimeter of the site.



(d) Sketch the graphs of $\sin(x)$ and $\sin(2x)$ between 0 and 2π . Hence solve the following equation $\sin(2x) = 0.866025$ for $0 \leq x \leq 360^\circ$.

5. (a) Find values of the first derivatives of the following at the given points:
- (i) $f(x) = (x^3 + 6x - 4)^3$ at $x = 2$
- (ii) $g(x) = (4x^2 - 7x)(9e^{3x})$ at $x = 0$
- (b) Given the function $y = x^3 - 5x^2 - x + 2$. Find the two turning points and specify if they are maximum or minimum points.
- (c) Determine the following integrals:
- (i) $\int \left(x^3 - 24x + 7x^{-1} - \frac{8}{x^3} \right) dx$
- (ii) $\int 9e^{2x} + \frac{26 + 9x^4 + 2x + 9}{x^2} dx$
- (d) Find the area under the curve $y = 9x^3 + 7x - 3$ between the values $x = 2$ and $x = 4$.