

**DUBLIN INSTITUTE OF TECHNOLOGY**

**First Year Engineering Entrance Examination 2010**

**In**

**MATHEMATICS**

**Attempt any 6 of the following 8 QUESTIONS**

**Time Allowed: 3 hours**

**Each question has 100 marks**

**All question carry equal marks**

**A table of formulae is attached**

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1. (a)  $(x-1)$ ,  $x$ ,  $(x+1)$  represent 3 *positive* numbers. The product is 8 times their sum. Find these 3 numbers (25)
- (b) Given that  $x=-2$  is a root of the equation  $x^3+6x^2+11x+6=0$ , find the other roots. (25)
- (c) Solve the simultaneous equations: (25)  
 $3x+y=7$   
 $xy+x^2=6$
- (d) A sum of € 11700 is to be shared among 3 people and the amounts are the first three terms of a geometric series. The smallest share is €1200. Find value of the largest share. (25)
2. (a) Find values of the derivatives of the following at the given points: (20 each)
- (i)  $y=\sqrt{3x^2+4}$  at  $x=2$
- (ii)  $y=\frac{x^2-1}{x+1}$  at  $x=1$
- (b) A company produces fruit juice cartons in the shape of a closed rectangular box. Each has base dimensions of  $2x$  cm by  $x$  cm with height of  $h$  cm. The capacity of each carton is to be 1030 cc.
- (i) Express  $h$  in terms of  $x$  (20)
- (ii) Show that the surface area of a carton  $A$  cm<sup>2</sup> is given by (20)  

$$A=4x^2+\frac{3090}{x}$$
- (iii) Find the value of  $x$  for which  $A$  will be a minimum and calculate this value of  $A$ . (20)
3. (a) A sum of €1000 invested on January 1<sup>st</sup> 2005 attracted an annual interest rate of 5%. By what factor does this sum increase each year and how much will it be worth on January 1<sup>st</sup> 2011. (25)
- (b) Given that  $(2x-1)$  is a factor of  $6x^3-kx^2-6x+8$ , find the value of  $k$ . (25)
- (c) Solve for  $x$ :
- (i)  $2 \ln(x-5)=4.16$  (10)
- (ii)  $\ln \frac{x}{x-5}=0.54$  (15)
- (d) The level of moisture in an object the amount after  $t$  hours is given by  $M=10e^{-0.4t}$  in grms. How much material is left after 2 hours and estimate how long it will take for the object to dry to a point where the moisture level falls to 0.1 grms? (25)

4. (a) Find  $a$  and  $b$  if  $(a+b) + i(a-b) = (2-i)^2 + i(4-5i)$ . (20)
- (b) Find the value of  $(1+2i)^6$  in both  $a+ib$  and polar form. (20)
- (c) Solve the complex equation:  $(x-2iy) - (y-ix) = 2+i$  (20)
- (d) Mark each of the following complex numbers on an Argand Diagram and express each in polar form:  $2i$ ,  $1+i$ ,  $-2+i$  (20)
- (e) Express  $\frac{5+i}{3-2i}$  in both  $a+ib$  and polar forms and find value of  $\left(\frac{5+i}{3-2i}\right)^8$  (20)
5. (a) Show  $\cos\left(\frac{\pi}{3}+x\right) + \cos\left(\frac{\pi}{3}-x\right) = \cos x$ . (25)
- (b) If  $\tan(x) = \frac{4}{3}$  and  $x$  is between  $180^\circ$  and  $360^\circ$ , find the values of  $\sin(x)$  and  $\cos(x)$ . (25)
- (c) From a point  $O$  on level ground, the angle of elevation to the top of a tower  $C$  is  $32^\circ$ . From a point  $A$ , 20 meters closer to the tower and on the same line with  $P$  and the base of the tower, the angle of elevation to the top of the tower is  $48^\circ$ . Find the height of the tower. (25)
- (d) Solve the equation  $\sin(2x+50^\circ) = 0.6$  for  $0 \leq x \leq 180^\circ$  (25)
6. (a) The points  $A(3,4)$  and  $B(9,12)$  are at the ends of a diameter of a circle. Find the equation of the circle (20)
- (b) Find the centre and radius of the circle  $x^2 + y^2 + 6x - 4y + 4 = 0$  (20)
- (c) Find the equation of the circle with centre  $(5,-1)$  which touches the  $y$  axis. (20)
- (d) Find the equation of the line that passes through both the point  $(1,0)$  and the point of intersection of the lines  $2x-y+6=0$  and  $10x+3y-2=0$  (20)
- (e) Given that  $5 + 3 \log_2 x = \log_2 y$ , show that  $y = 32x^3$ . (20)

7. (a) Evaluate 3 of the following integrals: (25 each)

(i)  $\int_1^3 \frac{2x}{(x+3)(x+1)} dx$

(ii)  $\int_2^5 \frac{1}{3x+5} dx$

(iii)  $\int_0^{\frac{\pi}{3}} x \cos x dx$

(iv)  $\int_{-1}^2 \frac{x}{(x^2+1)^2} dx$

- (b) Find the points where the line  $y=x$  cuts the curve  $y=4x-x^2$  and calculate the area enclosed by the curve and the line  $y=x$ . (25)

8. (a) Find the inverse of the matrix:  $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$  (20)

- (b) Put the following set of linear equations into matrix form and hence solve the set. (20)

$$3x + y = 8$$

$$5x + 2y = 13$$

- (c) If  $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$  and  $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ , show  $A^2 - 5A + 6I = 0$ . (20)

- (d) If  $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$  and  $X = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ , find  $k$  so that  $AX = kX$  (20)

- (e) Find the 2 values which satisfy the matrix equation: (20)

$$\begin{pmatrix} 1 & k \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ k \end{pmatrix} = 29$$