

**DUBLIN INSTITUTE OF TECHNOLOGY**

**First Year Engineering Entrance Examination 2013**

**In**

**MATHEMATICS**

**April 6<sup>th</sup> 2013**

**Attempt any 6 of the following 8 QUESTIONS**

**Time Allowed: 3 hours**

**Each question has 100 marks**

**All question carry equal marks**

**Maths Tables are available for use**

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**P.T.O.**

1 (a) Express  $2x^2+4x+12$  in the form  $a(x+b)^2 + c$  (25)

(b) Given that  $x+3$  is a factor of  $x^3 + ax^2 - 5x - 6$ , find  $a$ . (25)

(c) Solve the simultaneous equations:

$$\begin{aligned}x - y &= 4 \\x + 3y &= 20\end{aligned}$$

(25)

(d) An oil company drills a hole 100m deep. Estimate the cost of drilling if the cost is €35 for drilling the first metre with an increase in cost of €4 per metre in every succeeding metre. (25)

2. (a) Find values of the first derivatives of the following at the given points

(i)  $y = \sin(3x+50)$  at  $x=\pi/2$  (9)

(ii)  $f(x) = (2x^2 + 3x - 5)(\sin(x))$  at  $x=0$  (9)

(iii)  $g(x) = \sqrt{(x^2 + 2x - 1)}$  at  $x=2$  (7)

(b) Given the function  $y = x^3 + 2x^2 - x + 2$ . Find the two turning points and specify if they are maximum or minimum points. (25)

(c) A rocket launched from ground level rises  $x$  metres vertically upwards in  $t$  minutes and  $x = 200t - \frac{25}{2}t^2$ .

(i) Find the rate of change of  $x$  with respect to  $t$  when  $t=3$  min.

(ii) Find the highest point ( $x$ ) the rocket will reach and the time when this will happen. (25)

(d) Given the function  $y = \sqrt{2x^2 + kx - 1}$ ;  $x \neq 0$  and that  $\frac{dy}{dx} = 0$  for  $x = -1/2$ , find the value of  $k$ . (25)

3. (a) When a ball is dropped onto a hard surface it always bounces to a height equal to 0.8 of the height from which it was dropped. The ball is initially dropped from a height of 2m. How far does the ball travel before it stops bouncing?

(25)

- (b) Show  $x=2$  is a root of the cubic  $x^3 - 2x^2 - x + 2$ . Find other roots.

(25)

- (c) Solve for  $x$ :

(i)  $3 \log (2x-5) + \log (10) = 1$  (10)

(ii)  $\ln \frac{x}{x-3} = 0.2$  (15)

- (d) The amount of chemical in a reaction after  $t$  minutes is given by

$$M = 25 e^{-0.2t} \text{ in grams.}$$

How much material is left after 1 minute and estimate how long it will take for the amount of the chemical to reduce to 12 grams?

(25)

4. (a) Find  $a$  and  $b$  if  $a(3-4i) - b(1+2i) - 5 = 0$ .

(20)

- (b) If  $z_1 = 1 - 2i$  and  $z_2 = -1 + 2i$  find  $z_1 * z_2$  and  $\frac{z_1}{z_2}$ .

(20)

- (c) Show that  $z = 2 + i$  is a root of  $z^3 + z^2 - 15z + 25 = 0$ .

(20)

- (d) Express  $z = \frac{1+2i}{2-3i}$  in both  $a+ib$  and polar forms and find value of  $z^4$ .

(20)

- (e) Mark each of the following complex numbers on an Argand Diagram and express each in polar form:  $3-3i$ ,  $1+3i$ ,  $-5-i$ ,  $6i$ ,  $-2+4i$ .

(20)

5. (a) From a window 6 m above horizontal ground the angle of depression of the base of a higher building across the road is  $15^\circ$  and, from the same window, the angle of elevation of the top of the building is  $30^\circ$ . Calculate width of road and height of building.

(25)

(b)

- (i) Solve the triangle ABC given  $B=78^\circ$ ,  $AC=22.31\text{mm}$  and  $AB= 17.92\text{mm}$ .

(13)

- (ii) Sketch the graph for  $\cos(x)$  and  $\cos(2x)$ .

(12)

- (c) Solve the equation  $\cos(2x) = -0.5$  for  $0 \leq x \leq 180^\circ$

(25)

- (d) Prove that:

$$\cos(y - \pi) + \sin\left(y + \frac{\pi}{2}\right) = 0$$

(25)

- 6 (a) A circle has radius 5 and centre (1,2). Show that (4,6),(5,-1), (-3,5) and (-2,-2) all lie on the circle.

(25)

- (b) Find the equation of the circle with centre (2,6) which touches the y axis.

(25)

- (c) Find the equation of the line that passes through the point of intersection of the lines  $2x+2y-3=0$  and  $2x-y-2=0$  and is perpendicular to the line  $y+5x=2$ .

(25)

- (d) Solve for x:  $3x + 1 = \sqrt{2x + 10}$

(25)

7 (a) Evaluate 3 of the following integrals: (25 each)

(i)  $\int_1^2 \frac{4x+2}{(x^2+x-9)^3} dx$

(ii)  $\int_0^{\pi/2} \cos^2(x) dx$

(iii)  $\int \frac{2x-1}{(x+2)(x-3)} dx$

(iv)  $\int x \sin x dx$

(b) Find the area under the curve  $y = 4x^3 + 3x - 1$  between the values  $x=1$  and  $x=4$ . (25)

8. (a) Find  $k$  if  $B$  is the inverse of  $A$ :

$$A = \begin{pmatrix} k & -2 & k \\ -2 & 5k & -4 \\ k & -4 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 14 & 8k & 3 \\ 8k & 5 & 2 \\ 3 & 2 & k \end{pmatrix} \quad (25)$$

(b) Find the inverse of  $B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$  and use it to solve the following system of simultaneous equations:

$$\begin{aligned} 2x + y &= 8 \\ 3x + 2y &= 5 \end{aligned} \quad (25)$$

(c) Use Gaussian elimination to solve the following system of simultaneous equations:

$$\begin{aligned} 2x + y &= 8 \\ 3x + 2y &= 5 \end{aligned} \quad (25)$$

(d) Given the following matrices calculate:  $A*B$ ,  $B*A$  and  $A*C$ , if the operations are feasible. Explain your reasoning.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 3 \\ 4 & 2 \\ 12 & 6 \end{pmatrix} \quad C = \begin{pmatrix} 5 & 2 \\ 10 & 4 \end{pmatrix} \quad (25)$$